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IMPACT OF PARTICLES OF A GRANULAR MATERIAL ON A HARD SURFACE

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The recovery coefficient for the particle velocities of a number of granular materials of different sizes and shapes are determined experimentally.

In mathematical simulation of the motion of pulverized materials, it is sometimes necessary to calculate the motion of particles after their impact on the apparatus walls. In this, it is necessary to know the recovery coefficient of the normal and the tangential components of the particle velocity $k = w_{2n}/w_{1n}$ and $k_t = w_{2t}/w_{1t}$ and often their derivatives: the recovery coefficient of the total velocity $n = w_2/w_1$ and the ratio of angles $\varepsilon = \alpha_2/\alpha_1$. The problem of imperfectly elastic collision has been solved theoretically [1-3] only for spheres with a sufficiently large diameter, where the surface roughness can be neglected, while the recovery coefficient, which depends on the elastic characteristics of the particle and the surface, is either assigned or determined for an absolutely smooth surface. For the most important case — fine particles of irregular shape — a theoretical solution is apparently impossible, and the problem consists in obtaining reliable experimental relationships for certain materials with particles of any specific shape or slightly varying shapes. We shall present here the results of an experimental investigation of the rebound of particles with spherical and irregular shapes in relation to the particle dimension δ, the impact velocity w_1, and the impact angle α_1.

We used small glass balls with the density $\rho = 6600$ kg/m³ and small polystyrene balls ($\rho = 1080$ kg/m³), sorted by means of screens into the narrow particle size ranges δ = 40-45; 45-50; 50-56; 56-63; 63-70; 70-80; 80-90; 90-100; 100-125; 125-160 μm (glass) and 315-400; 400-500; 500-630 μm (polystyrene), and also calibrated steel balls for ball bearings with diameters of 680 and 1000 μm ($\rho = 7960$ kg/m³). For particles of irregular shape, we used coal (anthracite) particles in the same size ranges with $\rho = 1350$ kg/m³ (within 160-630 μm), corundum particles with $\rho = 3900$ kg/m³ (100-315 μm), and ferrochrome particles with $\rho = 6910$ kg/m³ (80-200 μm). The uniformity of the shapes of coal, corundum, and ferrochrome particles has been confirmed by means of reflected-light microscopy. The ratio of the maximum grain size to the minimum size is equal to 2-3. This can probably be explained by the homogeneity of these materials and the fact that the same method of crushing (in a ball mill) is used. The diameter of a sphere with the equivalent mass, determined by weighing 300-1000 particles, is used as the characteristic size of a particle size range. The characteristic size is equal to the arithmetic mean of sizes at the extremes of a range with a scatter of ±6%.

The particles are dropped vertically downward on an inclined ground steel plate at a velocity w_1 = 0.5-30 m/sec at an angle α_1 = 15-70° with respect to the normal to the plate at the point of impact. Their trajectories in the form of successive points are recorded on aerial photographic film, using illumination from an IFP-5000 flash bulb, supplied from a special source [4]. The flashing frequency is set within the range 200-500 Hz, depending on the impact velocity. The velocities w_1 and w_2 are determined with respect to the spacing between the points with an error of 2.5%, while the angles α_1 and α_2 are determined with respect to tangents to the trajectories at the impact point with an error of 3%. The values of k, k_t, n, and ε for each range of particle sizes of the material under investigation are obtained by statistically processing 20-30 trajectories for spheres and 30-50 trajectories for particles of irregular shape.

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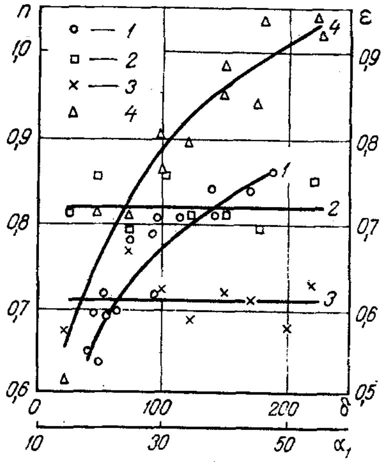

Fig. 1

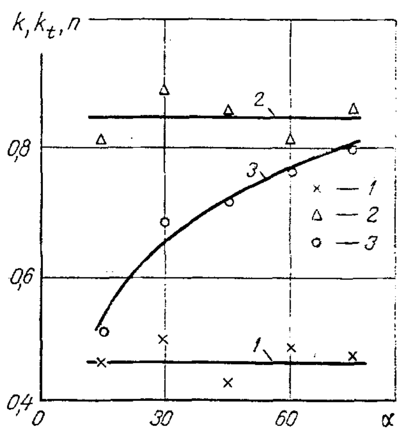

Fig. 2

Fig. 1. Dependence of the rebound characteristics on the dimension and the incidence angle of glass balls. 1) $n = f_1(\delta)$; 2) $n = f_2(\alpha_1)$ for $\delta = 142$ μm; 3) $n = f_3(\alpha_1)$ for $\delta = 60$ μm; 4) $\varepsilon = f_4(\alpha_1)$; α, deg.

Fig. 2. Dependence of the rebound characteristics on the incidence angle for coal particles. 1) $k = f_1(\alpha_1)$; 2) $k_t = f_2 \cdot (\alpha_1)$; 3) $n = f_3(\alpha_1)$.

According to [1], depending on the value of the parameter $\beta = \arctan[7f(1 + k)/2]$, there can exist two sets of conditions under which the final phase of sphere impact occurs: with or without slippage. If the impact angle is equal to $\alpha_1 < \beta$, the impact occurs without slippage, and vice versa. For $f = 0.35$ (the friction coefficient is determined experimentally), $k = 0.5$-0.8 and $\beta = 61$-$66°$, which exceeds the value of α_1 in the investigated range (15-50°); i.e., the impact of spheres ends without slippage in the case under consideration.

The impact characteristics are independent of w_1 in the investigated ranges. For glass spheres and $\alpha_1 = 15°$, n increases with δ (Fig. 1). This dependence is approximated by means of the following polynomial (δ, μm) with a root-mean-square error $\sigma_n = 0.035$:

$$h = 0.51 + 0.0036\delta - 0.0000095\delta^2. \tag{1}$$

For spheres with $\delta > 100$ μm, the values of k and n are in good agreement with those calculated on the basis of the assumptions made in [1]. In our opinion, the effect of the surface roughness of the plate is the most probable explanation for the reduction in k (and, consequently, n) for $\delta < 100$ μm.

If the recovery coefficient is expressed in terms of the velocity component which is normal to the plate protuberances, we readily obtain the approximate relationship

$$k \approx k_s\,[1 - (\Delta/\delta)^2], \tag{2}$$

where k_s is the recovery coefficient for an absolutely smooth plate surface, and Δ is the mean distance between protuberances (μm). Using a model-201 profilometer-profilograph, we found that $\Delta = 20$-30 μm at the point of particle impact. The values of k obtained for the above values are in satisfactory agreement with experimental data.

The value of ε is independent of δ, but it increases with α_1 (Fig. 1). The corresponding approximate expression ($\sigma_\varepsilon = 0.039$) has the following form (α_1, deg):

$$\varepsilon = 0.291 + 0.022\,\alpha_1 - 0.00019\,\alpha_1^2. \tag{3}$$

The value of n is virtually independent of α_1 (Fig. 1).

For polystyrene and steel spheres and $\alpha_1 = 15°$ the values of n, k, and k_t are approximately constant in a narrower range of δ (see Table 1, which also shows the root-mean-square deviations for k and k_t).

Large scatter values for n, ε, k, and k_t are characteristic for the rebound of particles of irregular shape, even if δ, α_1 = const. In order to obtain statistically reliable results,

TABLE 1. Recovery Coefficients of Various Materials

Material	k	k_t	n	ε	$\bar{\sigma}$, %
Coal	0,46	0,83	0,5	1,76	12,3
Corundum	0,325	1,115	0,408	2,83	10,1
Ferrochrome	0,328	1,123	0,432	2,84	12,1
Polystyrene	0,79	0,81	0,80	1,09	3,25
Steel balls	0,665	0,72	0,67	1,09	1,93

the minimum required number of trajectories (experiments) to be processed is determined so that the variance of the experimental values relative to the arithmetic mean does not decrease with a further increase in the number of experiments.

The basic characteristic of the rebound of particles with irregular shapes in comparison with spheres is that a sharp reduction in k and an increase in k_t (in a number of cases, $k_t > 1$; see Table 1) lead to a larger rebound angle. For all the investigated materials, $\varepsilon > 1$, while k, k_t, and n are independent of δ. For the coal size 315-400 μm, the values of k and k_t are constant as α_1 varies (Fig. 2), while the expression for n is approximated by the following polynomial ($\sigma_n = 0.02$, α_1, deg):

$$n = 0.344 + 0.013\,\alpha_1 - 0.000092\,\alpha_1^2. \quad (4)$$

A check of the reproducibility of results by means of Cochran's test, performed in order to estimate the reliability of the experimental data obtained, has shown that the condition for the uniformity of variances prevails. The values of the root-mean-square deviation σ for k and k_t, calculated with respect to the corresponding general variances, are given in Table 1; for glass balls, $\bar{\sigma} = 2.93\%$. An adequacy test has shown that the above dependences of the rebound characteristics on δ and α_1 correspond to the investigated process.

The data obtained in our experiments can be recommended for practical use in the mathematical simulation of dust-containing flows in various technological processes.

NOTATION

k, k_t, n, recovery coefficients of the normal, tangential, and total velocities, respectively; w_{1n}, w_{1t}, w_1, normal, tangential, and total impact velocities, respectively; w_{2n}, w_{2t}, w_2, same velocities for the rebound; α_1, angle between the impact velocity and the normal to the surface at the point of impact; α_2, same for the rebound velocity; δ, ρ, particle diameter and density, respectively.

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